

POGIL THE ACTIVITY SERIES ANSWERS Workbook

pogil the activity series answers are an essential resource for students and educators seeking to understand the reactivity of metals and the principles behind chemical reactions. The POGIL (Process Oriented Guided Inquiry Learning) activity series is designed to foster active learning through guided inquiry, helping learners grasp complex concepts related to the activity series of metals. This article provides a comprehensive overview of the activity series, offers detailed answers to typical POGIL activities, and explains how understanding these answers can enhance your grasp of chemical reactivity and metal displacement reactions.

Understanding the Activity Series of Metals

What Is the Activity Series?

The activity series of metals is a ranked list that arranges metals based on their reactivity with other substances, especially acids and water. Metals higher in the series are more reactive and can displace metals lower in the series from their compounds. This concept is fundamental in predicting the outcomes of various chemical reactions involving metals.

Purpose of the Activity Series

- To predict whether a metal will react with acids or water.
- To determine the feasibility of displacement reactions.
- To understand the relative reactivity of different metals.
- To assist in identifying safe handling and storage of metals.

Commonly Used Reactivity Series

The activity series typically includes metals such as:

- Potassium (K)
- Sodium (Na)
- Calcium (Ca)
- Magnesium (Mg)
- Aluminum (Al)

- Zinc (Zn)
- Iron (Fe)
- Lead (Pb)
- Copper (Cu)
- Mercury (Hg)
- Silver (Ag)
- Gold (Au)

Key Concepts in POGIL Activities on the Activity Series

Displacement Reactions

Displacement reactions occur when a more reactive metal displaces a less reactive metal from its compound. For example:

- Zinc reacting with copper sulfate: $\text{Zn} + \text{CuSO}_4 \rightarrow \text{ZnSO}_4 + \text{Cu}$

Reactions with Water and Acids

Metals react differently depending on their position in the activity series:

- Highly reactive metals (e.g., potassium, sodium) react vigorously with water.
- Moderately reactive metals (e.g., zinc, iron) react slowly or not at all with water but react with acids.
- Less reactive metals (e.g., copper, silver) usually do not react with acids or water.

Corrosion and Protection

Understanding the activity series helps in preventing corrosion:

- Sacrificial protection involves placing a more reactive metal (like zinc) to protect less reactive metals (like iron).

Common POGIL Activity Series Questions and Answers

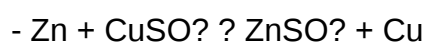
Question 1: Rank the following metals from most reactive to least reactive: magnesium, copper, zinc, and silver.

- Most reactive: Magnesium
- Zinc
- Copper
- Least reactive: Silver

Answer Explanation: Magnesium is higher in the activity series, meaning it reacts more readily with acids and water. Zinc is below magnesium but above copper and silver, which are much less reactive.

Question 2: Will zinc displace copper from copper sulfate? Explain your answer.

Answer: Yes, zinc will displace copper from copper sulfate because zinc is more reactive than copper according to the activity series. The reaction is:

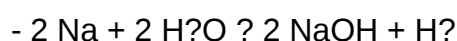


Question 3: Which metals will react with water? Provide examples from the activity series.

Answer: Metals above hydrogen in the activity series react with water:

- Potassium (K)
- Sodium (Na)
- Calcium (Ca)
- Magnesium (Mg)

Example: Sodium reacts vigorously with water:



Question 4: Why does gold not react with acids or water? Use the activity series to support your answer.

Answer: Gold is at the bottom of the activity series, indicating it has very low reactivity. Therefore, it does not react with acids or water because it is less reactive than hydrogen and cannot displace other elements from their compounds.

How to Use the Activity Series in Chemistry

Predicting Reactions

To predict if a displacement reaction will occur:

- Check the positions of the metals involved.
- If the metal you are adding is higher in the series than the metal in the compound, a reaction will likely occur.

Designing Experiments

- Select metals based on their position in the activity series.
- Observe reactions with acids, water, or other metal compounds.
- Record and analyze results to reinforce understanding.

Safety Considerations

- Highly reactive metals like potassium and sodium should be handled with care.
- Reactions involving these metals should be performed under supervision and with proper safety equipment.

Summary of Key Points for POGIL Activity Series Answers

- The activity series ranks metals based on reactivity.
- More reactive metals can displace less reactive metals from their compounds.
- Metals react differently with water and acids depending on their position.
- Gold and platinum are least reactive, often unreactive with acids and water.
- Understanding the activity series aids in predicting reactions, preventing corrosion, and designing chemical experiments.

Additional Tips for Mastering the Activity Series

Memorization Strategies

- Use mnemonic devices to remember the order of metals.

- Create flashcards with metal names and their positions.
- Practice predicting reactions regularly.

Real-World Applications

- Corrosion prevention in infrastructure.
- Metal extraction and refining processes.
- Designing batteries and electrochemical cells.
- Understanding environmental impacts of metal waste.

Practice Questions for Reinforcement

- Which metals can react with acids but not with water?
- How does the activity series explain the corrosion of iron?
- What is the significance of the activity series in recycling metals?

Conclusion

Understanding the **pogil the activity series answers** is fundamental to mastering concepts in chemistry related to metal reactivity. By exploring the principles behind the activity series, students can predict chemical reactions, understand displacement reactions, and appreciate the importance of metal reactivity in industrial processes and everyday life. Whether you're preparing for exams, conducting experiments, or simply seeking to deepen your understanding of chemistry, mastering the activity series through guided inquiry activities like POGIL can significantly enhance your knowledge and analytical skills.

Keywords: pogil activity series answers, metal reactivity, displacement reactions, activity series of metals, chemical reactivity, corrosion prevention, metal reactions with water, metal reactions with acids, POGIL activities, chemistry education

Pogil the Activity Series Answers: A Comprehensive Guide to Understanding and Using the Activity Series in Chemistry

Understanding the Pogil the Activity Series Answers is essential for students and educators who aim to master the concepts of reactivity in chemistry. The activity series, often presented in Pogil (Process Oriented Guided Inquiry Learning) activities, provides a systematic way to predict the outcomes of single replacement reactions and to understand the relative reactivity of metals and non-metals. This guide aims to demystify the activity series, explain how to interpret Pogil activities related to it, and provide strategies for mastering the answers effectively.

What is the Activity Series?

The activity series is a ranked list of elements (mainly metals) based on their ability to displace other elements from compounds, particularly in aqueous solutions. It helps chemists predict whether a certain single displacement reaction will proceed spontaneously.

Key Features of the Activity Series:

- Reactivity ranking: Elements are ordered from most reactive to least reactive.
- Displacement capability: More reactive metals can displace less reactive metals from their compounds.
- Standard reduction potentials: The series correlates with the elements' tendencies to gain electrons (be reduced).

The Role of Pogil Activities in Learning the Activity Series

Pogil activities are designed to facilitate inquiry-based learning. When applied to the activity series, these activities encourage students to analyze data, interpret trends, and develop reasoning skills related to chemical reactivity.

Typical Pogil Activity Structure:

- Initial exploration: Students observe and analyze experimental data or reaction patterns.
- Guided questions: Foster critical thinking about the relationships between element reactivity and their positions in the series.
- Application exercises: Students predict the outcomes of reactions based on the series and verify their reasoning.

How to Approach Pogil the Activity Series Activities

Mastering Pogil activities related to the activity series involves a strategic approach:

Step 1: Review the Activity Series Chart

Familiarize yourself with the standard activity series chart. Recognize the order of metals and how it relates to their reactivity.

Step 2: Understand Key Concepts

- Displacement reactions: Metal A displaces metal B if A is above B in the series.
- Reaction spontaneity: Reactions proceed spontaneously if the element doing the displacing is more reactive.

Step 3: Analyze Experimental Data

Look at the data provided in the activity:

- Which reactions occurred?
- Which did not?
- What patterns emerge?

Step 4: Answer Guided Questions Carefully

Reflect on the reasoning behind each answer:

- Why does a particular reaction happen or not?
- How does the position of an element in the series influence its reactivity?

Step 5: Practice with Variations

Try predicting the outcomes of reactions not explicitly covered in the activity to reinforce understanding.

Common Questions and Answer Strategies for Pogil the Activity Series

Below are typical questions encountered in Pogil activities and strategies for answering them effectively.

Question 1: Which metals can displace hydrogen from acids?

Answer approach:

- Refer to the activity series to identify metals above hydrogen.
- Metals like zinc, magnesium, and iron are above hydrogen and can displace it from acids.
- Metals below hydrogen (e.g., copper, silver) cannot.

Question 2: Why does a reaction occur between zinc and copper sulfate but not between copper

and zinc sulfate?

Answer approach:

- Look at the position of zinc and copper in the activity series.
- Zinc is above copper, so zinc can displace copper from its sulfate.
- Copper is below zinc, so it cannot displace zinc from its sulfate.

Question 3: Predict whether a given reaction will occur.

Answer approach:

- Check the relative reactivity of the metals involved.
- Confirm whether the metal attempting to displace is higher in the activity series.

Tips for Mastering the Activity Series and Pogil Answers

- Memorize the basic activity series: While not always necessary to memorize the entire series, knowing the general order helps in quick reasoning.
- Understand the underlying principles: Recognize that the activity series is based on oxidation-reduction tendencies.
- Practice with real examples: Use textbook problems, online quizzes, and lab experiments to reinforce your understanding.
- Use visual aids: Create flashcards or charts to visualize the series.
- Discuss with peers or instructors: Clarify doubts and explore different reasoning approaches.

Real-World Applications of the Activity Series

Understanding the activity series extends beyond academic exercises and into practical applications:

- Corrosion prevention: Selecting appropriate metals for construction to prevent rust.
- Electrochemical cells: Designing batteries based on the reactivity of metals.
- Metal extraction: Choosing suitable metals for reduction processes.

Conclusion

Mastering Pogil the Activity Series Answers is a fundamental step toward understanding chemical reactivity and predicting reaction outcomes. By familiarizing yourself with the structure of the activity series, practicing interpretation of Pogil activities, and applying critical reasoning strategies, you will enhance your comprehension of oxidation-reduction processes. Remember, the activity series is more than just a list—it's a powerful tool that unlocks the predictive power of chemistry in both academic and real-world contexts.

Empower your chemistry learning journey by engaging deeply with Pogil activities and the activity series. With practice and understanding, you'll develop the confidence to analyze reactions critically and apply your knowledge effectively.

Question What is the purpose of the POGIL activity series activity? The purpose is to help students understand the reactivity of metals by arranging them in a activity series based on their ability to displace or react with other substances. How do I determine the position of a metal in the activity series using POGIL activities? You determine the position by analyzing experimental results from displacement reactions and comparing how easily metals react with acids, water, or other solutions, as guided by the POGIL activity instructions. Are the answers to the POGIL activity series activity available online? Yes, various educational resources and teacher websites provide answer keys to POGIL activity series activities, but it's best to use them to understand concepts rather than just copying answers. What are common mistakes students make when completing the POGIL activity series activity? Common mistakes include misinterpreting experimental data, confusing the reactivity order of similar metals, or not following the experimental procedure carefully as outlined in the activity guide. How can I use the POGIL activity series to predict chemical reactions? By understanding the activity series, you can predict whether a metal will displace another from a compound or react with acids, based on their positions in the series. Is the POGIL activity series activity suitable for beginner chemistry students? Yes, it is designed to introduce students to the concept of reactivity and series of metals in a collaborative and inquiry-based manner suitable for beginners. Where can I find detailed answers or explanations for the POGIL activity series? You can find detailed explanations in teacher guides, educational websites, or textbooks that accompany the POGIL curriculum, and some online forums or study groups discuss these activities. How does understanding the activity series help in real-world applications? Understanding the activity series helps predict corrosion, metal extraction, and reactivity in industrial processes, making it essential for chemistry-related industries. Can I complete the POGIL activity series activity without prior knowledge of reactivity series? While prior knowledge helps, the activity is designed to guide students through the process of discovering the reactivity series through experimentation and guided inquiry. What should I do if I get stuck on the POGIL activity series question? Review the experimental data, revisit the activity instructions, collaborate with classmates, or consult your teacher for hints to help clarify the concepts and move forward.

Related keywords: activity series, pogil worksheet, reactivity series, metal reactivity, pogil activities, activity series chart, metal reactivity order, chemistry pogil, activity series answers, metal

displacement reactions

Theory And Application Of Infinite Series Dover B

Theory and application of infinite series dover b

Infinite series are fundamental constructs in mathematics that have profound implications across various scientific fields, including engineering, physics, computer science, and economics. The book Theory and Application of Infinite Series by Dover Publications (commonly referenced as Dover B) provides an in-depth exploration of the theoretical foundations and practical uses of infinite series. This article aims to delve into the core concepts, mathematical foundations, and diverse applications of infinite series, as well as highlighting key takeaways from Dover B.

Understanding Infinite Series

Infinite series are sums of infinitely many terms, typically expressed as:

$$S = \sum_{n=1}^{\infty} a_n$$

where (a_n) represents the n th term of the series. The primary questions surrounding infinite series involve convergence: does the sum approach a finite value as the number of terms increases?

Convergence and Divergence

- Convergence: An infinite series converges if the sequence of its partial sums approaches a finite limit.
- Divergence: If the partial sums do not settle to a finite value, the series diverges.

Understanding whether a series converges or diverges is crucial for its application in solving real-world problems.

Fundamental Concepts in Infinite Series (Dover B)

Dover B systematically introduces the fundamental concepts necessary for studying infinite series, including:

1. Geometric Series

A geometric series is of the form:

$$\sum_{n=0}^{\infty} ar^n$$

where a is the first term and r is the common ratio.

- Convergence Criterion: The series converges if $|r| < 1$

$$S = \frac{a}{1 - r}$$

- Applications: calculations of compound interest, population growth models, and signal processing.

2. p-Series and Comparison Tests

- p-Series: Series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$. Converges if $p > 1$, diverges otherwise.
- Comparison Test: Used to determine convergence by comparing with known series.

3. Alternating Series

Series with alternating signs, such as:

$$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

which converge under specific conditions, notably the Alternating Series Test.

Methods for Testing Series Convergence

Dover B emphasizes various techniques to determine the behavior of series:

1. The Ratio Test

Calculates:

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

- Converges if: $L < 1$
- Diverges if: $L > 1$
- Inconclusive if: $L = 1$

2. The Root Test

Considers:

$$L = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}$$

Similar criteria as the Ratio Test.

3. Integral Test

Applicable when $(a_n = f(n))$ is positive, decreasing, and continuous, relating the series to an improper integral:

$$\int_1^{\infty} f(x) \, dx$$

- Converges if: the integral converges
- Diverges if: the integral diverges

Power Series and Their Applications

Power series are infinite series of the form:

$$\sum_{n=0}^{\infty} c_n (x - a)^n$$

which are foundational in representing functions and solving differential equations.

Properties of Power Series

- Radius of Convergence: the range of (x) for which the series converges.
- Analytic Functions: many functions can be expressed as power series within their radius of convergence.

Applications of Power Series

- Approximate functions via Taylor and Maclaurin series.
- Numerical analysis and computation.
- Solving differential equations analytically.

Fourier Series and Signal Processing

Dover B dedicates significant attention to Fourier series, which express periodic functions as sums of sines and cosines:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

Applications in Engineering

- Signal analysis and filtering.
- Image processing.
- Heat transfer and wave propagation.

Key Concepts

- Fourier coefficients (a_n, b_n) determine the amplitude of each frequency component.
- Convergence properties depend on the function's smoothness.

Applications of Infinite Series in Real-World Problems

Infinite series underpin a multitude of practical applications across various disciplines:

1. Numerical Methods

- Series expansions enable approximation of complex functions.
- Used in algorithms for computing functions like exponential, logarithms, and trigonometric functions.

2. Physics and Engineering

- Analyzing oscillations, wave phenomena, and quantum mechanics.
- Modeling heat conduction via Fourier series.

3. Economics and Finance

- Present value calculations involving infinite cash flows.
- Modeling economic growth with geometric series.

4. Computer Science

- Algorithm analysis, especially recursive algorithms.
- Series approximations for computer graphics and simulations.

Advanced Topics and Modern Applications

Dover B also explores advanced topics that extend the classical theory of infinite series:

1. Summability Methods

Techniques like Cesàro summation allow assigning sums to divergent series in certain contexts.

2. Analytic Continuation

Extending functions defined by power series beyond their radius of convergence.

3. Zeta Function and Number Theory

Infinite series are central to understanding properties of prime numbers and complex analysis.

Conclusion: The Significance of Infinite Series

The theory and application of infinite series, as detailed in Dover B, serve as a cornerstone of mathematical analysis. Their versatility allows mathematicians and scientists to approximate, model, and solve complex problems across disciplines. Whether in theoretical physics, engineering, economics, or computational sciences, the concepts of convergence, series expansions, and their practical applications continue to be invaluable tools.

Understanding infinite series empowers practitioners to develop accurate models, perform precise calculations, and explore the deep structure of mathematical functions. Dover B's comprehensive treatment provides readers with both a solid theoretical foundation and practical insights, making it an essential resource for students, educators, and professionals engaged in applied mathematics.

Keywords: infinite series, convergence, divergence, geometric series, power series, Fourier series, numerical methods, applications, Dover B, mathematical analysis

Theory and Application of Infinite Series Dover B

Infinite series are fundamental constructs in mathematics that have profound implications across numerous scientific and engineering disciplines. Their study, particularly through the lens of Dover Publications' renowned texts, offers both a deep theoretical understanding and practical tools for tackling complex problems. This article explores the core principles behind infinite series, examines their theoretical underpinnings, and highlights their diverse applications, providing insights into why they are indispensable in modern mathematics.

Understanding Infinite Series: Foundations and Definitions

At its core, an infinite series is the sum of infinitely many terms arranged in a sequence. Formally, if $\{a_n\}$ is a sequence of real or complex numbers, then the infinite series is written as:

$$\sum_{n=1}^{\infty} a_n$$

$$S = \sum_{n=1}^{\infty} a_n$$

$$\sum_{n=1}^{\infty} a_n$$

The primary question mathematicians ask about such series is whether they converge or diverge. Convergence indicates that the partial sums approach a finite limit as the number of terms grows infinitely large, whereas divergence means the partial sums do not settle to a finite value.

Key Concepts:

- Partial Sums: $S_N = \sum_{n=1}^N a_n$
- Limit of Partial Sums: $\lim_{N \rightarrow \infty} S_N$

If this limit exists and is finite, the series converges to that limit; otherwise, it diverges.

Theoretical Foundations of Infinite Series

Convergence Tests and Criteria

Establishing whether an infinite series converges is foundational in analysis. Several tests aid in this determination:

- The Comparison Test: Compares the series with a known convergent or divergent series.
- The Ratio Test: Uses the limit of the ratio of successive terms to assess convergence.
- The Root Test: Considers the n th root of the absolute value of terms.
- The Integral Test: Connects the series to an improper integral to analyze convergence.
- Alternating Series Test: Specifically used for series with alternating signs, ensuring convergence if terms decrease in magnitude to zero.

Power Series and Radius of Convergence

A special class of infinite series is power series, expressed as:

$$\sum_{n=0}^{\infty} c_n (x - a)^n$$

where (c_n) are coefficients, and (a) is the center of the series. Power series are pivotal because they allow for the representation of functions as infinite sums, with convergence depending on the radius of convergence.

The radius of convergence (R) determines the interval where the series converges and can be found using the Root or Ratio test. Within this radius, the power series converges uniformly, enabling differentiation and integration term-by-term.

Summation of Infinite Series: Techniques and Formulas

Some series have well-known sums, such as geometric series:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1 - r}, \quad |r| < 1$$

Other series are more complex, involving special functions like the zeta function, or require advanced techniques such as generating functions or contour integration.

Applications of Infinite Series in Science and Engineering

The theoretical understanding of infinite series is not merely an academic pursuit; it forms the backbone of numerous practical applications.

- Signal Processing and Fourier Series

One of the most prominent applications is in signal analysis, where functions representing signals are expressed as sums of sinusoidal components via Fourier series:

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos nt + b_n \sin nt \right)$$

This decomposition allows engineers to analyze, filter, and reconstruct signals efficiently. Infinite series facilitate the transformation of complex, time-domain signals into frequency components, enabling technologies like telecommunications, audio processing, and image compression.

- Numerical Analysis and Approximation

Infinite series serve as powerful tools for approximating functions that are otherwise difficult to compute directly. Taylor and Maclaurin series expand functions into polynomials:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$$

This approach underpins many numerical algorithms in scientific computing, allowing for high-precision approximations of exponential, logarithmic, trigonometric, and other transcendental functions vital in simulations and modeling.

- Quantum Physics and Series Solutions

In quantum mechanics, solutions to differential equations like the Schrödinger equation often involve infinite series. Power series solutions help analyze potential wells, atomic orbitals, and scattering phenomena, where exact solutions are elusive. The use of series expansions simplifies complex wavefunctions into manageable computational forms.

- Economics and Financial Mathematics

Infinite series underpin models of compound interest, options pricing, and risk assessment. For example, the valuation of perpetuities involves the sum of an infinite geometric series, capturing the essence of continuous cash flows over an indefinite period.

Deep Dive: Dover's Contributions to Infinite Series Education

Dover Publications has long been a trusted source for classic mathematical texts, notably its comprehensive works on series and analysis. Their publications distill complex concepts into accessible formats, making the theory of infinite series approachable for students and professionals alike.

Key features of Dover's series-focused texts include:

- Rigorous yet clear explanations of convergence criteria
- Historical context of series development
- Extensive problem sets for mastery
- Coverage of special series such as Fibonacci, Bernoulli, and Fourier series

These resources have helped democratize advanced mathematical knowledge, fostering a deeper understanding of infinite series' role across disciplines.

Challenges and Frontiers in Infinite Series Research

While the theory of infinite series is well-established, ongoing research explores:

- Divergent Series: Methods like Borel summation extend the notion of summation beyond convergence, with implications in quantum field theory.
- Series Acceleration Techniques: Improving convergence rates for computational efficiency.
- Multidimensional Series: Extending concepts to multivariate functions and series in higher dimensions.
- Series in Non-Standard Analysis: Exploring series within alternative frameworks that challenge traditional notions of limits.

These frontiers continue to deepen our understanding and expand the utility of infinite series in solving cutting-edge problems.

Conclusion: The Enduring Significance of Infinite Series

The theory and application of infinite series, as expounded in Dover publications and beyond,

exemplify the elegance and power of mathematical abstraction. From analyzing signals and approximating functions to exploring the quantum realm and modeling economic phenomena, infinite series serve as a bridge between pure theory and tangible application.

Their study not only enriches our mathematical knowledge but also drives innovation across science and engineering. As research advances and computational methods evolve, the significance of infinite series is poised to grow, continuing to illuminate the complexities of the universe in elegant, infinite sums.

Question What is the primary focus of 'Theory and Application of Infinite Series' by Dover B? The book primarily explores the mathematical foundations, convergence criteria, and practical applications of infinite series in various fields such as analysis and physics. How does Dover B approach the convergence of infinite series? Dover B systematically introduces convergence tests like the comparison test, ratio test, root test, and alternating series test, providing rigorous proofs and intuitive explanations. Can Dover B's book be used for advanced undergraduate studies? Yes, the book is suitable for advanced undergraduates and beginning graduate students, offering a thorough treatment of infinite series with applications and exercises. What types of applications of infinite series are covered in Dover B? The book covers applications in mathematical analysis, approximation theory, Fourier series, and solutions to differential equations, demonstrating the practical significance of infinite series. Does Dover B include numerical methods related to infinite series? Yes, it discusses numerical techniques for summing series and approximating functions, emphasizing the computational aspects of infinite series. Are there any modern topics or recent developments in infinite series covered in Dover B? While the core focuses on classical theory, the book also touches on contemporary topics like power series, uniform convergence, and their role in functional analysis. How does Dover B differentiate itself from other texts on infinite series? Dover B combines rigorous theoretical treatment with practical applications, clear explanations, and a wide range of exercises, making complex concepts accessible. Is Dover B suitable for self-study on the topic of infinite series? Absolutely, the book's comprehensive coverage, detailed proofs, and exercises make it an excellent resource for self-learners interested in infinite series.

Related keywords: infinite series, mathematical analysis, convergence tests, power series, summation methods, series approximation, calculus, functional analysis, infinite sums, mathematical textbooks

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